Roll No.							

24-MA-41

M.Sc. IV SEMESTER [MAIN/ATKT] EXAMINATION
JUNE - JULY 2024

MATHEMATICS
Paper - I
[Functional Analysis - II]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 Let $T : H \rightarrow H$ be a bounded self adjoint operator, H is a complex Hilbert space than for

$B = (T^2)^{1/2}$, $T^+ = 1/2\ (B + T)$, $T^- = 1/2\ (B - T)$

and if $E : H \rightarrow Y\ = N(T^+)$

Prove the following -

i) B, T^+ and T^- commute with ; in particular
$BT = TB$, $T^+ T = T T^+$, $T^- T = T\ T^-$, $T^+ T^- = T^- \ T^+$

ii) E Commute with every bounded self adjoint linear operator that T commute with in particular
$ET = TB$ and $EB = BE$

OR

Let $T : H \rightarrow H$ be a bounded self - adjoint operator on a complex Hilbert space H and $\zeta = (E_\lambda)$ the corresponding spectral family. then there is a λ_0 belongs to the resolvent set p(T) of T if and only if there is a $\gamma > 0$ such that $e = (E_\lambda)$ is constant on the interval $[\lambda_0 - \gamma , \lambda_0 + \gamma]$.

Q. 2 State and prove Hellinger - Toeplitz theorem.

OR

With the help of an example define unbounded linear operators and their Hilbert - Adjoint operators in a complex Hilbert Space.

Q. 3 Let $T : D(T) \rightarrow H$ be a linear operator which are densely defined in a complex Hilbert space H. Suppose T is injective and its range R(T) is dense in H. Then show that T* is injective and
$(T^*)^{-1} = (T^{-1})^*$

OR

Prove that the spectrum $\sigma(T)$ of a self adjoint linear operator $T : D(T) \rightarrow H$ is real and closed ; here H is complex Hilbert space and D(T) is dense in H.

Q. 4 State and prove spectral theorem for unitary operators.

OR

Prove that the Cayley Transform

$U = (T - i\,I)\,(T + i\,I)^{-1}$

of a self adjoint linear operator

$T : D(T) \rightarrow H$ exists on H and is a unitary operator. Here $H \neq \{0\}$ is a complex Hilbert Space.

Q. 5 Define Multiplication Operator T. Show that a multiplication operator in Self - Adjoint.

OR

Define differentiation operator D and show that the differentiation operator is unbounded.

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